Tractable Bayesian Learning for Automated Design of Electromagnetic Structures

Ramesh Patel, *Student Member, IEEE*, Kallol Roy, Jaesik Choi, and Ki Jin Han, *Senior Member, IEEE*
School of Electrical and Computer Engineering, Ulsan National Institute of Science and Technology, Ulsan 44919, Korea, kjhan@unist.ac.kr

We propose a tractable Bayesian learning algorithm for Electromagnetic (EM) Structure design using high-dimensional data structure with Gaussian noise. Our learning method fastly converges on the hypothesis manifold and gives the optimum hypothesis. Our learning algorithm is scalable, and works on any general electromagnetic structure for automated design. We have tested our learning algorithm on the real EM data sheet and computed a theoretical upper bound for the uncertainty quantification. The fast convergence in the hypothesis space is done through various compositions of kernel functions through automatic statistician.

***Index Terms*—Automatic Statistician, Bayesian framework, Electromagnetic structure, Probabilistic statistics.**

I. Introduction

MULTI-dimensional electromagnetic (EM) structures have been applied to various electronic systems because they can smartly manipulate EM waves and reduce circuit sizes [1]. However, increased density of the EM structures results in narrower system margins with a larger number of system parameters to be optimized. Consideration of the overall dimensions with a large number of design variables requires a substantial computational burden, resulting in long design cycles [3].

To address the aforementioned problems, we propose a new tractable Bayesian learning for designing of EM structures on an automated manner. The method of learning from randomly sampled minimum data sets using Bayesian Models for giving maximum-likelihood hypothesis (h_{ML}) with error control. The fast convergence of picking the h_{ML} out of myriads of hypotheses is a difficult problem to solve. To overcome this issue, we use the method of automatic statistician. Apart from this, we have verified our learning algorithm for a $n \times n$ metamaterial structure and computed a theoretical upper bound for uncertainty quantification.

II. Proposed Approach

A. Methodology

The proposed method of tractable learning using Bayesian statistics on EM data structure (EMD) is described in Fig. 1. The EMD is first fed to the Bayesian engine hypothesis in search of the optimal hypothesis h_{ML}. The mean-squared error is used as a metric to search the h_{ML} in the space of hypotheses. Our method of computing h_{ML} is tractable, as the probability of picking the wrong hypothesis is bounded. The data from our simulations are corrupted with general noise model. The uncertainty of the noise model in the simulation data is not inhibitive to our tractable learning model, though we have used Gaussian noise for searching the h_{ML}. As the hypothesis space is huge (but finitely countable) and we do not have any structural information *a priori*, the choosing of one hypothesis over other, is equally likely.

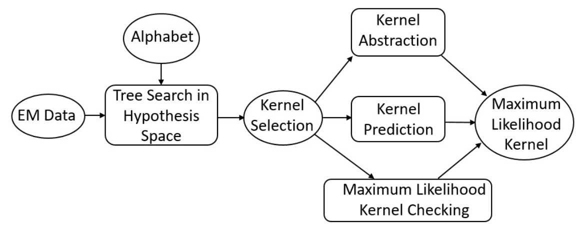

Fig. 1: Proposed methodology.

B. Bayesian Learning Model

We write the Bayesian learning as:

$$P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)} \tag{1}$$

where h is the hypothesis that defines the functional mapping, $D = \{x_i^L, d_i\}$ is a data set obtained from full-wave simulations of an EM structure, x_i is the geometry parameter of the EM structure, d_i is the output EM response, and L is the dimensionality [2].

Maximum likelihood hypothesis h_{ML} is then given by:

$$h_{ML} = \text{argmax } P(D \mid h) \tag{2}$$

$$= \text{argmax } \prod_{i=1}^{N} P(D \mid h) \tag{2a}$$

Assuming the Gaussian noise $\sim N(0, \sigma^2)$ with identically distributed (i.i.d.) assumptions and taking natural log on both sides of the above equation, the h_{ML} is given by:

$$h_{ML} = \text{argmax } \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(di - h(x_i^L))^2}{\sigma^2}} \tag{3}$$

$$\approx \text{argmax } \sum_{i=1}^{N} -\frac{1}{2}\frac{(d_i - h(x_i^L))^2}{\sigma^2} \tag{3a}$$

$$\approx \text{argmax } \sum_{i=1}^{N} (d_i - h(x_i^L))^2 \tag{3b}$$

C. *Fast Converging on Hypothesis Tree Structure*

We get the bounded mean squared error by greedy searching the h_{ML} from the hypothesis tree structure shown in Fig. 2, with the automatic statistician. The optimal kernel is selected by the Bayesian maximum-likelihood using the compositional application of rules. A grammar is created to efficiently infer the components and estimate predictive maximum-likelihood. Then a greedy search algorithm is used choosing the decomposition structure from data by random sampling of all models. The automatic statistician method then finds the optimum kernel structure.

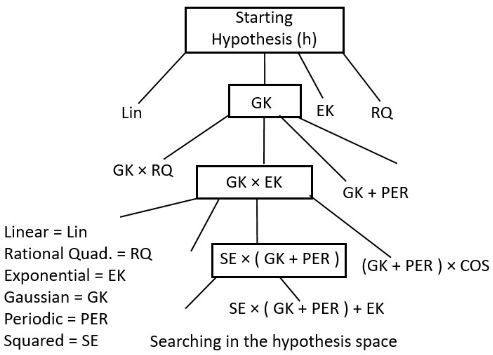

Fig. 2: Hypothesis Space with Automatic Statistician.

D. *Uncertainty Quantification in Electromagnetic Data*

We propose uncertainty quantification (UQ) using Chebychev inequality. Using the Chebyshev inequality framework, we computed the upper bounds on uncertainties of the EMD. The problems of extremizing probabilities of error bound, are subject to the constraints imposed by the assumptions and information of the data structure. The computed data variance values of output parameters (f_r and μ_{eff}) are 1.895 and 0.002, respectively.

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}\left\{d_i - h(x_i^L)\right\}^2 \tag{4}$$

$$P[(d - \langle h(x^1, x^2, x^3, x^4)\rangle)^2] \leq \frac{1}{k^2}\left\langle (d - \langle h(x^1, x^2, x^3, x^4)\rangle)^2\right\rangle \tag{5}$$

where x^1, x^2, x^3, and x^4 are dataset notations of geometric parameters, k is permissible percentage error of hypothesis.

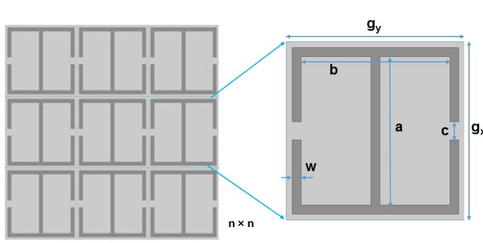

Fig. 3: A schematic diagram of $n \times n$ MTM prototype with notations a = b = x_i^1, c = x_i^2, g_x = g_y = x_i^3, w = x_i^4 for $i = N$ number of input dataset.

E. *Results and Discussions*

The proposed method is applied to an $n \times n$ metamaterial structure shown in Fig. 3. We find the h_{ML} by using various combinations of kernels (in Fig. 2) and comparing the MSE. In the Fig. 4 and 5 MSE values with respect to various hypothesis functions are shown for topmost 35 random input geometric parameter samples of $n \times n$ MTM prototype (in Fig. 3). The h_{ML} computed for frequency and permeability is the periodic hypothesis and the minimum MSE are 0.04 and 0.002 respectively. The real EM data is matching quite well with hypothesis data and also justifies the convergence of our tractable learning algorithm.

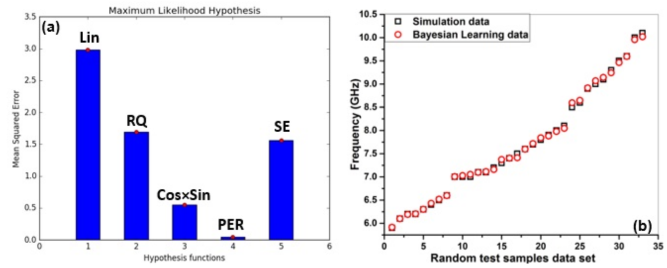

Fig. 4: For f_r (a) MSE of various hypotheses functions (b) Machine learning output and simulated data for topmost samples (x_i^L).

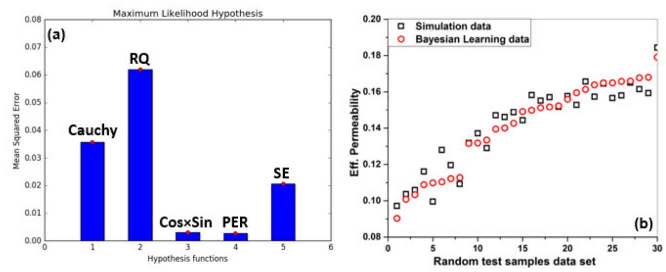

Fig. 5: (a) MSE of various hypotheses functions for μ_{eff} (b) Machine learning output and simulated data for topmost samples at 10 GHz.

III. Conclusion

We propose a new method of tractable learning using Bayesian statistics on EM data using Bayesian Models for giving maximum-likelihood hypothesis through automatic statistician. We successfully mapped (4-in-1) output function $f(x_i^L)$ for input data sets $\{x_i^L\}$ (L = 4) over d_i. Our learning architecture is scalable in time and space complexity and is solely driven by the input data only. We have tested convergence of our learning algorithm on the real EM data and propose a theoretical upper bound for the uncertainty quantification.

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